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AERODYNAMIC EFFECTS IN ISOTOPE SEPARATION BY GASEOUS DIFFUSION

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Abstract—The turbulent flow of an isotopic mixture in a porous-walled pipe is considered in the presence of suction through the wall. A simple model is formulated for the evaluation of aerodynamic effects on the separation efficiency. The predictions of the model are found to compare very favourably with experiment. In the limit of small suction velocities results obtained by other investigators for diffusion in a turbulent stream are recovered.

NOMENCLATURE

A,	suction rate, $A = \lvert\bar{w}\rvert/U$;
a_*,	ideal separation factor, equation (16);
C,	effusion velocity at pore mouth;
d,	pipe diameter;
D,	diffusion coefficient;
j,	molecular number density flux;
k_B,	Boltzmann's constant;
l,	characteristic length, equation (12);
m,	molecular mass;
n,	molecular number density;
r,	richness in light component, $r = X/(1-X)$;
Re,	Reynolds number based on the average velocity and pipe diameter;
S,	separation efficiency;
S_l,	wall separation efficiency, equation (17);
Sc,	Schmidt number, $Sc = \nu/D$;
Sh,	Sherwood number, equation (14);
T,	absolute temperature;
$\mathbf{u}$,	velocity vector of individual components;
$\bar{\mathbf{u}}$,	center-of-mass velocity vector, equation (7);
u_*,	friction velocity;
U,	average flow velocity in pipe;
$\bar{w}$,	center-of-mass velocity component in the radial direction, taken constant;
X,	molar fraction of lighter component;
X'_A,	enriched molar fraction of the lighter component which would be obtained if the molar fraction at the wall were X^A;
X^*,	molar fraction at the wall necessary to obtain the enriched molar fraction X' in an ideal separative process, equation (A2);
y,	distance from the wall in the radial direction;
y^+,	dimensionless distance from the wall in the radial direction, equation (3);
Z,	aerodynamic efficiency, equation (2).

Greek symbols

γ,	dimensionless phenomenological constant equation (19);
δ,	dimensional thickness of diffusion–convection layer;
δ^+,	dimensionless thickness of diffusion–convection layer;
ν,	kinematic viscosity;
ρ,	fluid density;
σ,	porosity of the wall;
τ_w,	wall shear stress.

Subscripts

A,	axial values, taken equal to the inlet values;
h,	heavy component;
l,	light component.

Superscripts

0,	values at pipe wall;
$'$,	enriched values.

1. INTRODUCTION

ISOTOPIC separation by gaseous diffusion is based on the well-known fact that in a gas mixture the fraction of molecules hitting a surface element of the container per unit time is inversely proportional to the square root of the molecular mass. In the hypothesis of a Maxwellian velocity distribution with zero mean this quantity can be readily computed with the result (see e.g. [1])

$$j = \tfrac{1}{4}n^0(8k_BT/\pi m)^{1/2}. \qquad (1)$$

Here j denotes the number of molecules hitting the wall of the container per unit area and unit time, n^0 is the molecular (number) density in the vicinity of the wall, T is the absolute temperature, m is the molecular mass, and k_B is Boltzmann's constant. Thus, if a small opening (pore) is made in the wall, relatively more molecules of the lighter than of the heavier species will tend to effuse out of the container in a given time. In the industrial application of this principle, the isotopic mixture flows in a series of separative units each of which consists essentially of a pipe, the wall of which is made of a porous material. The pipe wall is surrounded by a low pressure chamber into which the lighter molecules effuse at a faster rate than the heavier ones, and from which therefore a gas richer in the lighter species than that entering the pipe can be withdrawn.

In the steady state a concentration gradient in the radial direction is necessarily present in the pipe because a greater number of the lighter than of the heavier molecules need be transported to the wall per unit time to compensate for the different effusion rates. As a consequence the ratio of the effusive fluxes through the porous barrier will be different from the one that would be found if the composition at the wall were equal to the inlet composition. The effect of this gradient can be quantitatively described in terms of the aerodynamic efficiency Z defined by [2]

$$Z = \frac{X' - X^A}{X'_1 - X^A}, \tag{2}$$

where X is the molar fraction of the lighter component, $X = n_l/(n_l + n_h)$, with the subscripts l and h indicating the lighter and heavier species respectively. In equation (2) the prime denotes values at the pore exit and the superscript A denotes values on the pipe axis. The quantity X'_1 is the molar fraction at the pore exit which would be obtained if the molar fraction at the wall, X^0, were equal to X^A. It is clear that, as a consequence of the presence of the radial gradient, $X^0 < X^A$, from which follows that $X'_1 > X'$, and hence that $Z < 1$.†

In the present study we describe a simple model for the calculation of the aerodynamic efficiency for conditions of turbulent flow in a separative unit.

The model contains one undetermined parameter the value of which is chosen by comparison with experimental data. We show that, with the value of the constant determined in this way, our model is in very good agreement with experiment in the range $5 \times 10^3 \lesssim Re \lesssim 10^5$, where Re is the Reynolds number based on the pipe diameter.

We may observe that any mass flux through the pores will perturb the Maxwellian velocity distribution used to obtain equation (1) and will result in a collective, non-separative flow in addition to the molecular effusion [3,4]. This effect can be reduced using pores the cross section of which have linear dimensions smaller than the mean free path of the gas molecules. For this reason the early separation plants were forced to operate at low pressures, and therefore at such small flow rates that the flow was laminar. This situation prompted the analysis of the laminar version of the problem under consideration which has been the object of a number of studies [5–14]. The improvement of manufacturing techniques for porous barriers has made possible the use of higher pressures and turbulent flow, for which much less information is available, at least in unclassified sources [2,15]. It will be clear from the following that the effect of turbulence is highly beneficial in that it reduces the thickness of the diffusion boundary layer near the wall which, as was observed by Caldirola [6] and other authors [12], has adverse effects on the separative efficiency.

2. DESCRIPTION OF THE MODEL

Fully developed turbulent flow in a pipe is, at least at the descriptive level, a quite well understood phenomenon [16]. The fluid is in a fully turbulent state of motion over most of the cross section of the pipe. The influence of the wall can be described in terms of the dimensionless distance y^+ defined by

$$y^+ = yu_*/\nu, \tag{3}$$

where y is the dimensional distance from the wall, ν the kinematic viscosity, and $u_* = (\tau_w/\rho)^{1/2}$ the friction velocity expressed in terms of the fluid density ρ and wall shear stress τ_w. The region $0 \leqslant y^+ \lesssim 5$ is occupied by the laminar sublayer, and transition to the turbulent core takes place over the intermediate range $5 \lesssim y^+ \lesssim 70$.

In view of the very effective mixing properties of turbulent flow the radial concentration gradient will be much stronger in the boundary layer than farther from the wall. In addition, the small radial velocity $\bar{v}$ induced by the effusion process will be appreciable only in the neighborhood of the wall, where it is of the same order as the local average axial velocity or greater. These considerations motivate our major assumptions, namely that the concentration gradient becomes negligible at a distance $y^+ = \delta^+$ from the wall, and that the radial velocity $\bar{v}$ is constant in the layer $0 \leqslant y^+ \leqslant \delta^+$ and zero elsewhere (see e.g. [17]). These assumptions are modeled after those made by Caldirola in his analysis of the laminar case [6] for which however they appear to be somewhat less justified than here.

It is clear that this schematization is rather crude. A detailed analysis would undoubtedly show that the parameter δ^+ (provided that it can be rigorously defined at all) depends on the concentrations, the Reynolds number, the effusion rate, and possibly other quantities. It will be shown below, however, that the dependence on these factors is not too critical, and that our hypothesis can be a valid first approximation.

According to (3) the dimensional thickness of the convection–diffusion layer is given by

$$\delta = \delta^+ \nu/u_*. \tag{4}$$

For Reynolds numbers smaller than 10^5 an expression for u_* can be obtained from the Blasius resistance formula [16] which, when inserted into (4), results in

$$\delta = 5.03\delta^+ d Re^{-7/8}, \tag{5}$$

where d is the pipe diameter and $Re = Ud/\nu$ is the Reynolds number based on the average velocity U. For higher values of the Reynolds number Prandtl's universal law of friction may be used in place of Blasius'; equation (5) however is sufficient for the present purposes. The parameter δ^+ is the only free constant of our model, and it will be determined by comparison with experimental data.

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† The aerodynamic efficiency is usually defined by an equation similar to (2) in which the richness $r = X/(1-X)$ appears in place of the molar fraction. Usually one has $X \ll 1$, so that $r \simeq X$ approximately. In view of the very slight composition variation produced by a single separative unit we may consider X^A constant, and equal to the inlet value of the molar fraction of the lighter component.

If compressibility effects are neglected the diffusion equation for the lighter molecular species in steady conditions can be written as [18]

$$\nabla\cdot[n_l\mathbf{u}-D\nabla n_l]=0, \tag{6}$$

with a similar equation for n_h. Here D is the diffusion coefficient of the lighter molecules in the heavier ones and $\bar{\mathbf{u}}$ is the center-of-mass velocity of the fluid particles defined by

$$\bar{\mathbf{u}}=X\mathbf{u}_l+(1-X)\mathbf{u}_h, \tag{7}$$

where $\mathbf{u}_l$ and $\mathbf{u}_h$ are the velocity fields of the lighter and of the heavier molecular species. A straightforward estimate of the order of magnitude of the terms involved shows that the derivatives in the axial direction in equation (6) are small compared with those in the radial one. With the neglect of the former therefore we may integrate the equation directly with the result

$$n_l\bar{w}-D\frac{\partial n_l}{\partial y}=-j_l^0, \tag{8a}$$

where $\bar{w}$ is the radial component of the velocity $\bar{\mathbf{u}}$. The constant of integration has been put equal to the molecular effusive flux at the wall, $-j_l^0$, which is negative with our choice of the y axis away from the wall. A similar result holds for the heavier molecules, namely

$$n_h\bar{w}-D\frac{\partial n_h}{\partial y}=-j_h^0. \tag{8b}$$

With the assumption of uniform suction velocity $\bar{w}$, equations (8) can be readily integrated from the wall, $y=0$, to $y=\delta$. Because of our hypothesis on the absence of concentration gradients beyond $y=\delta$ the boundary conditions are $n_l=n_l^A$, $n_h=n_h^A$ at $y=\delta$, where the superscript A indicates the values on the pipe axis which, for small suction rates, can be taken as the inlet values. In this way the number density n_l^0 at the pipe wall can readily be computed with the result

$$n_l^0=(n_l^A+j_l^0/\bar{w})\exp(-\bar{w}\delta/D)-j_l^0/\bar{w}, \tag{9}$$

and similarly for n_h^0. This is not a final result however because the effusive flux j_l^0 is a function of n_l^0; an example of this relationship is given by equation (1) for the ideal case. In general the connection between j_l^0 and n_l^0 may be assumed to be a linear function, except when a strong non-separative collective flow is present [2–4].

We therefore set

$$j_l^0=\sigma C_l n_l^0, \tag{10}$$

where the constant C_l, which is dimensionally a velocity, is given by $C_l=\frac{1}{4}(8k_BT/\pi m_l)^{1/2}$ in the ideal case (1), and by more complicated expressions when the effects of retrodiffusion, finite pore length, etc. are taken into account [2–4]. The quantity σ is the porosity of the wall, defined as the total pore mouth area per unit area.

With (10) equation (9) can be solved for n_l^0 with the result

$$n_l^0=\frac{\exp(-\bar{w}\delta/D)}{1-\sigma\dfrac{C_l}{\bar{w}}[\exp(-\bar{w}\delta/D)-1]}n_l^A, \tag{11a}$$

A corresponding expression holds for n_h^0,

$$n_h^0=\frac{\exp(-\bar{w}\delta/D)}{1-\sigma\dfrac{C_h}{\bar{w}}[\exp(-\bar{w}\delta/D)-1]}n_h^A. \tag{11b}$$

Notice that in these equations the suction velocity $\bar{w}$ is negative. Expressions similar to (10) were obtained by Caldirola for the laminar case [6].

3. COMPARISON WITH EXPERIMENT

It is convenient to introduce a length l characteristic of the radial transport process through the definition

$$l=(X^A-X^0)\left[\left.\frac{\mathrm{d}X}{\mathrm{d}y}\right|_{y=0}\right]^{-1}, \tag{12}$$

where X denotes the molar fraction of the lighter component. With the aid of equations (8) and (11) we obtain the following explicit expression for this quantity

$$l=\frac{D}{|\bar{w}|}\left[\exp\left(\frac{\delta|\bar{w}|}{D}\right)-1\right], \tag{13}$$

where use has been made of the relation

$$\sigma(n_l^0C_l+n_h^0C_h)=-\bar{w}(n_l^0+n_h^0),$$

which is a consequence of (7), (10) and of the assumed constancy of $\bar{w}$. An interesting feature of the relation (13) is its independence from the effusion velocities C_l, C_h. In terms of the Schmidt number $Sc=\nu/D$ and of the suction rate $A=|\bar{w}|/U$ this equation can be put in the form

$$Sh^{-1}=\frac{l}{d}=(AReSc)^{-1}\times[1-\exp(-5.03\delta^+ARe^{1/8}Sc)] \tag{14}$$

in which the expression (5) for δ has been used. Notice that l/d is the inverse of the Sherwood number for the present problem. The quantity l cannot be measured directly, but it can be related to the aerodynamic efficiency Z defined in (2). It is shown in the Appendix that the following relation holds:

$$Z=\frac{1-|\bar{w}|l/D}{[1+S_l(a_*-1)l|\bar{w}|/D](1-X^A)}, \tag{15}$$

where a_* is the ideal separation factor expressed in terms of the molecular masses of the heavy and light isotopes by [1]

$$a_*=(m_h/m_l)^{1/2}. \tag{16}$$

S_l is the wall separation efficiency defined by [2]

$$S_l=\frac{X'-X^0}{X^0}\frac{X^*}{X'-X^*}, \tag{17}$$

where X^* is the molar fraction at the wall necessary to obtain the enriched molar fraction X' in an ideal separative process. By definition therefore S_l is less than 1, and it accounts for the non ideal features of the separation. The connection of this quantity with the separation efficiency of Present and De Bethune [4] is discussed in the Appendix.

As shown by Mordchelles-Regnier [2] the aerodynamic efficiency can be obtained from the measurement of the concentration of the lighter isotope in the enriched isotopic mixture. The value of l can then be deduced from (15).

The most significant prediction of our model is that the quantity δ^+ is independent of Reynolds number or suction rate. In order to test this prediction we shall rely on the data of Mordchelles-Regnier [2][†] who reduced his results in terms of $Sh^{-1} = l/d$. In this process we shall also be able to determine the value of the quantity δ^+.

Table 1 shows a selection of data of Mordchelles-Regnier's with the measured values of Re, A, Z and Sh^{-1}; in the last column the value of δ^+ deduced from (14) is shown. A certain scatter in the results is evident. In order to determine whether this scatter was due to experimental error or to a systematic dependence of δ^+ on Re or A neglected in our model we tried a least-square fit of the values of δ^+ by an expression of the form $\delta^+ = kRe^aA^b$, with k, a, b constants. From the data of the table the following result was obtained

$$\delta^+ = 31.1Re^{-0.022}A^{0.067}.$$

Notice that the factor $Re^{-0.022}$ goes from 0.83 to 0.78 for values of the Reynolds number ranging between 5×10^3 and 10^5. Similarly the factor $A^{0.067}$ ranges between 0.86 and 1.08 for $0.1 \times 10^{-3} \leqslant A \leqslant 3 \times 10^{-3}$. The extremely modest variation of these factors over such wide ranges suggests that the scatter observed in the last column of Table 1 is of purely experimental origin, and lends a strong support to the validity of the model presented in Section 2. The average value of δ^+ determined from the data is

$$\delta^+ = 15.8. \tag{18}$$

In Table 2 we present the data with $Re < 5000$. These data were not considered before because they do not correspond to fully developed turbulent flow in the separation unit. Somewhat surprisingly, however, it appears that the present model gives a satisfactory fit of these data also. When the least square fit is carried out on the data of both Tables 1 and 2 one finds $\delta^+ = 27.2Re^{-0.004}A^{0.073}$, a result that lends itself to the same interpretation given above for the previous one.

Finally, the remaining data of Mordchelles-Regnier are shown in Table 3 with the corresponding δ^+ values. These data have not been considered above because they appear affected by an obvious large experimental error, as can be deduced by comparison with the data of Table 1 corresponding to similar values of Re and A.

The least-square fit on all the data of Tables 1, 2 and 3 gives the result $\delta^+ = 57.6Re^{-0.051}A^{0.123}$.

Table 1. The data from [2] used to obtain the result (18)

Re	$A \times 10^3$	Z	$Sh^{-1} \times 10^2$ experimental	δ^+
5087	1.127	0.722	4.37	15.0
5178	1.684	0.735	4.07	14.8
5220	2.203	0.678	4.11	15.8
5277	2.975	0.580	4.25	17.8
5380	1.846	0.717	4.12	15.9
5546	1.675	0.739	4.07	15.9
6124	2.174	0.648	3.85	17.3
7385	1.216	0.791	3.33	16.2
7415	1.216	0.798	2.87	13.8
7592	1.338	0.784	2.89	14.4
8050	1.714	0.692	3.20	17.9
8299	2.421	0.622	2.75	16.6
8414	2.394	0.619	2.73	16.6
8420	1.723	0.696	3.15	18.4
8516	2.710	0.592	2.63	16.5
8599	2.239	0.683	2.43	14.5
8632	1.762	0.687	3.18	19.2
8668	3.373	0.519	2.49	16.8
8766	2.536	0.598	2.29	14.1
8847	1.370	0.754	2.88	16.8
9074	3.051	0.534	2.49	17.2
9387	2.365	0.613	2.62	17.6
9404	2.274	0.620	2.69	18.1
10406	0.841	0.842	2.48	15.8
10520	0.892	0.822	2.26	14.5
10944	0.932	0.821	2.41	16.2
11055	0.795	0.853	2.22	14.8
11370	0.799	0.789	2.19	14.9
11452	1.158	0.777	2.33	16.8
11582	1.546	0.708	2.17	16.3
11973	1.203	0.736	2.56	19.6
12036	1.804	0.661	2.23	18.0
12228	0.965	0.859	2.19	16.3
12404	2.305	0.613	2.02	17.5
13091	1.586	0.689	2.16	18.5
13360	0.695	0.869	1.90	14.8
16378	0.612	0.864	1.85	17.3
16500	0.559	0.892	1.63	15.2
16509	0.800	0.856	1.56	14.8
16787	0.779	0.847	1.63	15.8
16906	1.105	0.777	1.65	16.7
17003	1.845	0.672	1.58	17.3
17379	1.213	0.745	1.66	17.5
17474	1.043	0.794	1.50	15.4
17482	1.540	0.696	1.55	16.9
18364	0.626	0.886	1.32	13.5
18846	1.062	0.794	1.49	16.5
19379	0.439	0.933	1.11	11.6
19520	0.447	0.907	1.38	14.7
20174	0.438	0.952	1.44	15.8
26900	0.362	0.878	1.14	16.0
32551	0.935	0.807	0.92	16.3
34768	0.252	0.935	0.94	16.4
48542	0.270	0.950	0.58	13.5
50142	0.201	0.952	0.65	15.5
50615	0.528	0.853	0.68	17.1
52170	0.353	0.927	0.48	11.9
52250	0.911	0.911	0.62	15.6
53107	0.266	0.957	0.63	15.9
56956	0.362	0.957	0.51	13.8
63178	0.467	0.908	0.36	10.7
69193	0.472	0.880	0.37	11.9
100763	0.241	0.933	0.23	10.1
107621	0.101	0.969	0.38	17.5

4. DISCUSSION AND CONCLUSIONS

The assumptions that we have made in Section 2 amount to the hypothesis of the existence of cut off

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[†] These data have been taken on an argon isotopic mixture with the lighter isotope, of atomic weight 36, present in the concentration of 0.34%.

Table 2. The data from [2] corresponding to $Re < 5000$

Re	$A \times 10^3$	Z	$Sh^{-1} \times 10^2$ experimental	δ^+
2716	3.796	0.559	7.03	16.7
2860	3.914	0.551	6.53	16.3
2866	3.595	0.572	6.66	16.4
3281	4.179	0.582	6.56	20.7
3379	2.663	0.642	5.55	14.7
3390	3.164	0.592	5.95	16.7
3641	2.683	0.621	5.92	17.3

Table 3.†

Re	$A \times 10^3$	Z	$Sh^{-1} \times 10^2$ experimental	δ^+
31 216	0.258	0.976	0.33	5.1
56 882	0.208	0.961	0.29	7.6
95 205	0.149	0.983	0.18	7.4
100 892	0.102	0.981	0.71	31.3
101 267	0.148	0.987	0.17	7.4
103 278	0.197	0.752	0.16	7.1
104 903	0.247	0.951	0.21	9.5
109 548	0.115	0.993	0.12	5.6
109 826	0.206	0.953	0.15	7.0

†The remaining data of [2] not considered in the present study because of the probable presence of large experimental error. Notice that according to [2] data corresponding to values of Z near 1 can be affected by substantial imprecisions.

distances for the effect of the effusion velocity and of the concentration gradient. The comparison with experimental data made in the preceding Section substantiates the validity of this assumption over a large range of Reynolds numbers and suction rates. A physical basis for this result can be sought in the fact that the radial velocity is extremely small (of the order of a thousandth of the mean axial velocity), so that any appreciable amount of turbulence is sufficient to render its effect negligible. This interpretation is supported by the fact that the constant δ^+ is found to have a value of the order of 16, which is not too far from the outer boundary of the laminar sublayer. We may also remark that we expect the laminar sublayer to become somewhat thicker as an effect of the suction through the porous wall, in view of the induced dampening of the turbulent eruptions from the boundary layer [19]. Another significant feature of the present model is the use of Blasius' law. Suction is known to induce strong modifications in the axial velocity in the laminar sublayer but its effect on the outer portions of the boundary layer is much smaller [20]. Since in the present model we neglect velocities in the axial direction and we use explicitly only results relative to the transition sublayer, it is expected that Blasius' law applies in its original form.

One question left open by our theory is that of the dependence of the value of δ^+ from the axial concentration and the Schmidt number. Concerning the first point it appears likely that such a dependence exists as soon as the concentration of the lighter component becomes of the order of 5% or greater, because the radial gradients to be expected in these situations will be much greater and less readily wiped out by turbulent transport. Let us recall, however, that in very many practical situations of industrial interest one deals with concentrations smaller than 1%, for which our theory appears to be quite adequate.

A dependence on the Schmidt number $Sc = \nu/D$ is also most likely present, because the quantity δ^+ is determined by a competition of momentum and mass transport. We expect δ^+ to increase with the mass diffusion length, because then appreciable gradients extend to a longer distance from the wall, and to decrease with the momentum diffusion length, because the greater this quantity the deeper into the buffer layer turbulent fluctuations from the main stream penetrate. This qualitative argument leads to postulate as a first approximation, an inverse dependence of the cut off distance with the square root of the Schmidt number, $\delta^+ = \gamma Sc^{-1/2}$. These speculations can be put on a firmer basis observing that a power series expansion of the exponential in equation (14) yields

$$\frac{l}{d} \simeq 5.03 \gamma Sc^{-1/2} Re^{-7/8}. \tag{19}$$

We can compare this expression with the Gilliland correlation [21,22] quoted by Mordchelles-Regnier [2] as describing accurately his data,

$$\frac{l}{d} = 43.48 Sc^{-0.44} Re^{-0.83}. \tag{20}$$

(This expression represents a fit of data relative to vapor diffusion from liquid films in an air stream.) Notice that, since $7/8 = 0.875$, the exponent of the Reynolds number is quite close to our result (5). Furthermore, over the limited range of Schmidt numbers tested, the exponent 0.44 differs negligibly from 1/2, and is thus compatible with the above considerations. These slight differences may safely be imputed to experimental error. The comparison between the numerical constants in (19) and (20) is somewhat complicated by the slight difference in the exponents which leads to the relation

$$\gamma = 8.64 Sc^{0.06} Re^{0.045}. \tag{21}$$

For $Re = 10\,000$, which is approximately in the middle of the range investigated by Gilliland, and $Sc = 0.75$, (21) gives $\gamma = 12.8$ to be compared with the value 13.1 obtained from (17) and (18). The agreement improves if, instead of considering the average (18) of all the entries of Table 1 we select only those corresponding to $A \leqslant 10^{-3}$, which should limit the error caused by the series expansion of our result (14). In this way we obtain an average $\delta^+ = 14.7$, to which corresponds $\gamma = 12.7$.

Introducing the additional Schmidt number dependence just determined into equation (14) we can put it in the following final form

$$Sh^{-1} = (A Re Sc)^{-1}[1 - \exp(-\gamma Sc^{1/2} A Re^{1/8})],$$

where γ is a numerical constant. Its value deduced from (18) is

$$\gamma = 13.1,$$

while that obtained from the Gilliland formula is

$$\gamma = 12.7.$$

On the basis of the available experimental evidence it is not possible to discriminate between these two values.

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APPENDIX

We give here a derivation of equation (15) of the text. The standard relation between the effusive fluxes and the enriched molar fraction at the other side of the porous wall X' is [1]

$$\frac{X'}{1-X'} = \frac{j_l^0}{j_h^0}.$$

Making use of equations (10) and (7), and rearranging we obtain

$$\sigma C_l = -(X'/X^0)\bar{w},$$

which can be combined with equation (8a) after division by (n_l+n_h) to give, at the wall,

$$X^0\bar{w} - D\left.\frac{\partial X}{\partial y}\right|_{y=0} = X'\bar{w}. \tag{A1}$$

Notice that this procedure is legitimate because the pressure in the boundary layer can be taken to be constant. The enriched molar fraction X' can be related to the molar fraction at the wall X^0 through the wall separation efficiency S_l defined by (17) in which X^* is given by the well-known expression [1]

$$X^* = \frac{X'}{a_* - X'(a_*-1)}. \tag{A2}$$

The wall separation efficiency S_l is related to the separation efficiency S defined by Present and de Bethune [4] by

$$S_l = SX^*/X^0,$$

which can be put into the form

$$S_l = \frac{S}{1+(a_*-1)(1-X')(1-S)} \simeq \frac{S}{1+(a_*-1)(1-S)},$$

since $X' \ll 1$ in the situations of present concern. An explicit expression for the quantity S has been obtained by Present and de Bethune [4] and a simpler, approximate formula has been given by Massignon [2].

Elimination of X^* between (17) and (A2) results in

$$X' = \frac{1+S_l(a_*-1)}{1+S_l(a_*-1)X^0}X^0 \simeq [1+S_l(a_*-1)]X^0, \tag{A3}$$

where the small quantity $S_l(a_*-1)X^0$ has been neglected in the denominator. With this relationship it is possible to obtain the following expression for the aerodynamic efficiency (2)

$$Z = \frac{[1+S_l(a_*-1)]X^0 - X^A}{S_l(a_*-1)X^A(1-X^A)}, \tag{A4}$$

where the same approximation made in (A3) has been used. It is now possible to substitute (A3) into (A1) and to make use of the definition of l, equation (12), to eliminate $[\mathrm{d}X/\mathrm{d}y]_{y=0}$. In this way the following expression for X^0 is obtained

$$X^0 = [1+S_l(a_*-1)l|\bar{w}|/D]^{-1}X^A$$

which, substituted into (A4), gives (15). Notice that this derivation is independent of the particular model used in this paper, and therefore of general validity.

EFFETS AERODYNAMIQUES DANS LA SEPARATION ISOTOPIQUE PAR DIFFUSION GAZEUSE

Résumé—*Cet article étudie l'écoulement turbulent d'un mélange d'isotopes dans une conduite à paroi* poreuse lorsqu'il y a aspiration à travers la paroi. Les auteurs proposent un modèle simple pour l'évaluation des effets aérodynamiques sur le pouvoir de séparation. Les résultats obtenus à partir de ce modèle sont en bon accord avec ceux de l'expérience. Dans le cas des petites vitesses d'aspiration, on retrouve également les résultats obtenus par d'autres chercheurs pour la diffusion dans un écoulement turbulent.

Zusammenfassung—Es wird die turbulente Strömung einer Isotopenmischung in einem porösen Rohr unter der Saugwirkung der Wand betrachtet. Für die Berechnung des Einflusses von aerodynamischen Effekten auf die Trennleistung wird ein einfaches Modell angegeben. Die Voraussagen des Modells stimmen sehr gut mit dem Experiment überein. Im Bereich kleiner Sauggeschwindigkeiten werden die Ergebnisse anderer Forscher für Diffusion in einer turbulenten Strömung betrachtet.

АЭРОДИНАМИЧЕСКИЕ ЭФФЕКТЫ ПРИ РАЗДЕЛЕНИИ ИЗОТОПОВ ПОСРЕДСТВОМ ГАЗОВОЙ ДИФФУЗИИ

Аннотация — Рассматривается турбулентное течение изотопной смеси в трубке с пористыми стенками при наличии отсоса через стенку. Предложена простая модель для оценки влияния аэродинамических эффектов на интенсивность разделения изотопов. Найдено, что модельные расчеты хорошо согласуются с экспериментом. В пределах небольших скоростей отсоса получено подтверждение данных других исследователей по диффузии в турбулентном потоке.